A Distributed Game-Theoretic Solution for Power Management in the Uplink of Cell-Free Systems

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***Abstract*—This paper investigates cell-free massive multiple input multiple output systems with a particular focus on uplink power allocation. In these systems, uplink power control is highly non-trivial, since a single user terminal is associated with multiple intended receiving base stations. In addition, in cell-free systems, distributed power control schemes that address the inherent spectral and energy efficiency targets are desirable. By utilizing tools from game theory, we formulate our proposal as a non-cooperative game, and using the best-response dynamics, we obtain a distributed power control mechanism. To ensure that this power control game converges to a Nash equilibrium, we apply the theory of potential games. Differently from existing game-based schemes, interestingly, our proposed potential function has a scalar parameter that controls the power usage of the users. Numerical results confirm that the proposed approach improves the use of the energy stored in the battery of user terminals and balances between spectral and energy efficiency.**

***Index Terms*—Cell-free systems, game theory, Nash equilibrium, potential game, power control, radio resource allocation.**

I. Introduction

Radio resource allocation is a major issue in the design of modern mobile networks. In interference-limited systems, for example, power control plays an indispensable role in managing interference, ensuring proper signal strength at the intended receivers and saving energy. Recognizing the scarcity of the energy resource and the growing worldwide energy concern, efficient power control solutions have definitely become a key requirement for the continued success of wireless systems [1]–[3]. Especially related to the uplink and given the ever-increasing growth in mobile subscriptions, an efficient power allocation strategy is important to reduce energy demands and battery consumption. By mitigating interference levels, power control has also the advantage of providing a more uniform throughput among users. Furthermore, an optimized energy consumption contributes to reducing environmental impacts, e.g., heat dissipation and electronic pollution [4].

However, the power management in cellular networks is a fairly complex problem. In general, efficiently controlling power usage with multiple interfering users may lead to non-polynomial time (NP)-hard problems, and in these cases obtaining optimal solutions is extremely challenging. Normally, within the power allocation framework, the main difficulties in finding alternative solutions are the performance coupling among the users as well as their inherently selfish behaviors. Consequently, a good solution needs to deal with the interactions among several independent users with contrasting interests. In this context, game theory provides a natural framework for developing mechanisms when many individuals with conflicting interests interact. Therefore, it is a promising approach to study interactions among contending users in order to seek feasible and practically viable solutions [5], [6]. Indeed, there has been growing interest in adopting game-theoretic methods to propose alternative solutions in mobile communications, see, e.g., [7]–[12].

In [7] and [8], the authors focused on an uplink power control game-based solutions for orthogonal frequency division multiple access (OFDMA) systems and cognitive radio networks, respectively. To address the problem of minimizing the sum of the mean square errors (MSEs), power control schemes for the uplink of massive multi-user multiple input multiple output (MU-MIMO) systems were proposed in [9]–[11]. More specifically, considering block fading channels, [9] and [10] relied on game theory to optimize the pilot-to-data power ratio assuming single and multi-cell cases, respectively. Likewise, a game-based approach to controlling the pilot and data power levels was presented in [11] while considering more realistic auto-regressive channels. On the other hand, in more recent network architectures such as cell-free, game-theoretic approaches to radio resource allocation have not been widely explored in the literature, mainly related to the uplink. Recently, in [12], a power control game was proposed for cell-free massive multiple input multiple output (MIMO) systems, but the authors focused on the downlink.

Inspired by the above discussion, this paper considers the problem of power control for the uplink of cell-free massive MIMO systems. Due to the inherent competitive nature of the multi-user and user-centric environment, we use a game-theoretic framework and model the problem as a strategic non-cooperative game, which can often provide feasible and convenient alternatives for a distributed implementation. Specifically, we use novel payoff functions based on an adapted signal-to-interference ratio (SIR) expression. More importantly, different from existing works, our solution is designed as a parameterized potential game for which the existence and uniqueness of a Nash equilibrium is ensured. Thereby, we show that the proposed power control achieves efficient solutions with respect to different network objectives such as sum-rate maximization, max-min fairness or power consumption minimization.

II. Network Model

We consider a cell-free massive MIMO system consisting of K single-antenna user equipments (UEs) and L access points (APs) equipped with N antennas grouped in the sets $\mathcal{K}$ and $\mathcal{L}$, respectively. The APs and UEs are deployed randomly in a wide area without boundaries. A central processing unit (CPU) connects with the APs via a backhaul network.

Particularly, we analyze a cell-free massive MIMO system operating in time division duplex (TDD) mode with a pilot phase for channel estimation and a data transmission phase. Each coherence block is divided into τ_p channel uses for uplink pilots, τ_u for uplink data and τ_d for downlink data such that $\tau_\mathrm{c} = \tau_\mathrm{p}+\tau_\mathrm{u}+\tau_\mathrm{d}$. The channel between AP l and UE k is denoted as $\mathbf{h}_{kl} \in \mathbb{C}^N$ and $\mathbf{h}_k = \left[\mathbf{h}_{k1}^\mathrm{T}, \dots, \mathbf{h}_{kL}^\mathrm{T}\right]^\mathrm{T} \in \mathbb{C}^{NL}$ is the collective channel from all APs. In each coherence block, an independent realization from a correlated Rayleigh fading distribution is drawn as $\mathbf{h}_{kl} \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \mathbf{R}_{kl})$, where $\mathbf{R}_{kl}$ is the spatial correlation matrix describing the spatial property of the channel and $\beta_{kl} = \mathrm{tr}(\mathbf{R}_{kl})/N$ is the large-scale fading coefficient that describes pathloss and shadowing [13], [14]. The Gaussian distribution models the small-scale fading whereas the positive semi-definite correlation matrix $\mathbf{R}_{kl}$ describes the large-scale fading, including shadowing, pathloss, spatial channel correlation and antenna gains. Given that the APs are spatially distributed in the system, the channel vectors of different APs are independently distributed, i.e., $\mathbb{E}\left\{\mathbf{h}_{kl'} \left(\mathbf{h}_{kl}\right)^\mathrm{H}\right\} = \mathbf{0}$ when $l' \neq l$. The collective channel is distributed as $\mathbf{h}_k \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \mathbf{R}_k)$, where $\mathbf{R}_k = \mathrm{diag}\left(\mathbf{R}_{k1}, \dots, \mathbf{R}_{kL}\right) \in \mathbb{C}^{NL\times NL}$ is the block-diagonal spatial correlation matrix [13].

We define a set of block-diagonal matrices $\mathbf{D}_k = \mathrm{diag}\left(\mathbf{D}_{k1}, \dots, \mathbf{D}_{kL}\right) \in \mathbb{C}^{NL\times NL}$, $k \in \mathcal{K}$, where $\mathbf{D}_{il} \in \mathbb{C}^{N\times N}$, $i \in \mathcal{K}$ and $l \in \mathcal{L}$ is the set of diagonal matrices, determining which AP antennas may transmit to which UEs. More specifically, the n-th diagonal entry of $\mathbf{D}_{il}$ is 1 if the n-th antenna of AP l is allowed to transmit and to decode signals from UE k, and 0 otherwise. Based on the definition of the set of matrices $\mathbf{D}_{il}$, we define a matrix $\mathbf{A} \in \mathbb{R}^{K\times L}$ specifying the AP selection, where $A_{k,l} = 1$ if AP l is allowed to transmit and to decode signals from UE k, i.e., if $\mathrm{tr}\left(\mathbf{D}_{kl}\right) > 0$, and 0 otherwise. For the conciseness of mathematical descriptions, we denote by $\mathcal{M}_k = \{l \,|\, A_{k,l} = 1, l \in \mathcal{L}\}$ the subset of APs serving UE k. Meanwhile, $\mathcal{D}_l = \{k \,|\, A_{k,l} = 1, k \in \mathcal{K}\}$ is the subset of UEs served by AP l.

A. Uplink pilot-based channel estimation

We consider that there are τ_p mutually orthogonal τ_p-length pilots, with τ_p being a constant independent of K. Let $\mathcal{S}_t \subset \mathcal{K}$ be the subset of UEs assigned to pilot t. When the UEs in $\mathcal{S}_t$ transmit, the received signal $\mathbf{y}_{tl}^\mathrm{pilot} \in \mathbb{C}^N$ at AP l is

$$\mathbf{y}_{tl}^\mathrm{pilot} = \sum_{i\in\mathcal{S}_t} \sqrt{\tau_\mathrm{p}\rho_i}\mathbf{h}_{il} + \mathbf{n}_{tl}, \tag{1}$$

where ρ_i is the transmit power of UE i, τ_p is the processing gain, and $\mathbf{n}_{tl} \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \sigma^2\mathbf{I}_N)$ is the thermal noise. Note that, since we assume a massive access scenario with a large number of UEs, i.e., $K > \tau_\mathrm{p}$, several UEs share the same pilot as shown in (1), leading to pilot contamination.

For estimating the channels, the classic minimum mean squared error (MMSE) criterion has been recurrently employed in the literature. The MMSE estimate of $\mathbf{h}_{kl}$ for UE $k \in \mathcal{S}_t$ is $\hat{\mathbf{h}}_{kl} = \sqrt{\tau_\mathrm{p}\rho_k}\mathbf{R}_{kl}\boldsymbol{\Psi}_{tl}^{-1}\mathbf{y}_{tl}^\mathrm{pilot}$, where $\boldsymbol{\Psi}_{tl} = \sum_{i\in\mathcal{S}_t}\tau_\mathrm{p}\rho_i\mathbf{R}_{il} + \sigma^2\mathbf{I}_N$ is the correlation matrix of (1). The estimated channel $\hat{\mathbf{h}}_{kl}$ and estimation error $\tilde{\mathbf{h}}_{kl} = \mathbf{h}_{kl} - \hat{\mathbf{h}}_{kl}$ are independent vectors distributed as $\hat{\mathbf{h}}_{kl} \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \mathbf{B}_{kl})$ and $\tilde{\mathbf{h}}_{kl} \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \mathbf{C}_{kl})$, where $\mathbf{B}_{kl} = \mathbb{E}\left\{\hat{\mathbf{h}}_{kl}\hat{\mathbf{h}}_{kl}^\mathrm{H}\right\} = \tau_\mathrm{p}\rho_k\mathbf{R}_{kl}\boldsymbol{\Psi}_{tl}^{-1}\mathbf{R}_{kl}$ and $\mathbf{C}_{kl} = \mathbb{E}\left\{\tilde{\mathbf{h}}_{kl}\tilde{\mathbf{h}}_{kl}^\mathrm{H}\right\} = \mathbf{R}_{kl} - \mathbf{B}_{kl}$.

B. Uplink data transmission

During the uplink data transmission, AP l receives the signal $\mathbf{y}_l \in \mathbb{C}^N$ from all UEs, as

$$\mathbf{y}_l = \sum_{k\in\mathcal{K}} \mathbf{h}_{kl}s_k + \mathbf{n}_l, \tag{2}$$

where $s_k \in \mathbb{C}$ is the signal transmitted from UE k with power ρ_k and $\mathbf{n}_l \sim \mathcal{NC}(\mathbf{0}, \sigma^2\mathbf{I}_N)$. However, since only a subset of APs take part in the signal detection, the estimate of s_k is:

$$\begin{aligned}\hat{s}_k &= \sum_{l\in\mathcal{L}} \mathbf{v}_{kl}^\mathrm{H}\mathbf{D}_{kl}\mathbf{y}_l \\ &= \mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{h}_k s_k + \sum_{i\in\mathcal{K}\setminus\{k\}} \mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{h}_i s_i + \mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{n},\end{aligned} \tag{3}$$

where $\mathbf{v}_{kl} \in \mathbb{C}^N$ is a receive combining vector of AP l for UE k, $\mathbf{v}_k = \left[\mathbf{v}_{k1}^\mathrm{T}, \dots, \mathbf{v}_{kL}^\mathrm{T}\right]^\mathrm{T} \in \mathbb{C}^{NL}$ denotes the collective of these combining vectors and $\mathbf{n} = \left[\mathbf{n}_1^\mathrm{T}, \dots, \mathbf{n}_L^\mathrm{T}\right]^\mathrm{T} \in \mathbb{C}^{NL}$ collects all the noise vectors.

Preferably, for large-scale networks, it is more interesting to direct the main computational tasks to the APs in a distributed way and, thus, avoid overloading the CPU. Therefore, instead of sending $\{\mathbf{y}_{tl}^\mathrm{pilot}\}_{\forall t}$ and $\mathbf{y}_l$ to the CPU, each AP l locally selects the combining vector $\mathbf{v}_{kl}$ and then it preprocesses its signal by computing local estimates of the data as $\hat{s}_{kl} = \mathbf{v}_{kl}^\mathrm{H}\mathbf{D}_{kl}\mathbf{y}_l$. Next, the local estimates of all APs that serve UE k are sent to the CPU for final estimate of s_k, which is given by $\hat{s}_k = \sum_{l\in\mathcal{L}}\hat{s}_{kl}$. We utilize the *use-and-then-forget* bound to obtain the achievable spectral efficiency (SE).

Lemma 1. [13], [14]. *An achievable uplink SE for UE k is*

$$\mathrm{SE}_k = \frac{\tau_\mathrm{u}}{\tau_\mathrm{c}}\log_2(1 + \mathrm{SINR}_k), \tag{4}$$

where

$$\mathrm{SINR}_k = \frac{\rho_k\left|\mathbb{E}\{\mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{h}_k\}\right|^2}{\sum\limits_{i\in\mathcal{K}}\rho_i\mathbb{E}\left\{\left|\mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{h}_i\right|^2\right\} - \rho_k\left|\mathbb{E}\{\mathbf{v}_k^\mathrm{H}\mathbf{D}_k\mathbf{h}_k\}\right|^2 + \sigma^2\mathbb{E}\left\{\left\|\mathbf{D}_k\mathbf{v}_k\right\|^2\right\}}. \tag{5}$$

In general, any combining vector that depends on the local channel estimates and statistics can be used in the signal-to-interference-plus-noise ratio (SINR) expression (5), but the expectations in it cannot be computed in closed form for any set of values $\{\mathbf{v}_{kl}\}_{\forall k,l}$. With simpler combining vector structures, such as maximum ratio combining (MRC), i.e., when $\mathbf{v}_{kl} = \hat{\mathbf{h}}_{kl}$, it is possible to obtain closed form expressions. Nevertheless, the performance of MRC is quite limited, and significant performance gains can be obtained when using combining vectors also with distributed structures but based on the MMSE criterion, such as local partial MMSE (LP-MMSE) combining [13], [14], whose combining vector $\mathbf{v}_{kl}^{\text{LP-MMSE}}$ is:

$$\mathbf{v}_{kl}^{\text{LP-MMSE}} = \rho_k \left(\sum_{i \in \mathcal{D}_l} \rho_i \left(\hat{\mathbf{h}}_{il} \hat{\mathbf{h}}_{il}^{\text{H}} + \mathbf{C}_{il} \right) + \sigma^2 \mathbf{I}_N \right)^{-1} \hat{\mathbf{h}}_{kl}. \quad (6)$$

However, when using $\{\mathbf{v}_{kl}^{\text{LP-MMSE}}\}_{\forall k,l}$ the SINR expression in (5) can only be computed via Monte Carlo simulations [13].

III. Resource Allocation

In this section, we describe the radio resource management employed in the network that consists of two parts. In the first part, we simply adopt the algorithm for joint initial access, pilot assignment, and cluster formation proposed in [13, See Section V-A]. Then, in the second part, differently from [13], we pay special attention to power control and propose a game-theoretic model of the interactions among users assuming a distributed management framework, which is presented in details in the following subsections.

A. Game Theoretic Approach

In the context of game theory, the players are considered as entities with the ability of observation and reaction. For the proposed game model, in order to mitigate potential interference levels in the uplink and obtain a suitable data power profile, the players are the users themselves. We define the proposed non-cooperative game $\mathcal{G}$ as $\mathcal{G} = \left\{\mathcal{K}, \{\mathcal{P}_k\}_{\forall k}, \{\mu_k(\rho_k, \boldsymbol{\rho}_{(-k)})\}_{\forall k}\right\}$, where $\mathcal{K}$ is the set of UEs, i.e., a finite set of players. For a given UE k, $\mathcal{P}_k = [\rho_{\min}, \rho_{\max}]$ is a finite set of available strategies or actions, where $\rho_{\min} > 0$ and $\rho_{\max} \leq P_{\max}$ with $P_{\max}$ being the maximum uplink data power. In the context of the proposed game, the data power value $\rho_k \in \mathcal{P}_k$ denotes the strategy chosen by UE k and $\boldsymbol{\rho}_{(-k)}$ denotes the strategies of all the UEs other than UE k. Therefore, $\boldsymbol{\rho} = (\rho_k, \boldsymbol{\rho}_{(-k)}) = [\rho_1, \cdots, \rho_k, \cdots, \rho_K]^{\text{T}} \in \mathbb{R}^K$ represents the profile of data powers of all UEs, i.e., a power allocation strategy for the system. Moreover, $\mu_k(\rho_k, \boldsymbol{\rho}_{(-k)})$: $\boldsymbol{\Upsilon} \to \mathbb{R}$, is a real-valued utility/payoff function where, $\boldsymbol{\Upsilon} = \mathcal{P}_1 \times \mathcal{P}_2 \times \cdots \times \mathcal{P}_K$ is the strategy space. Note that, for every chosen strategy by UE k, the power profile $(\rho_k, \boldsymbol{\rho}_{(-k)})$ is associated with a payoff, i.e., $\mu_k(\rho_k, \boldsymbol{\rho}_{(-k)})$. Thus, the payoff function quantifies the preferences of each UE to a given action, provided the knowledge of others' actions.

Typically, a non-cooperative game is a procedure where each player will selfishly choose an action that improves its own utility function given the current strategies of the other players. Then, a key issue when designing a game is the choice of the payoff function. Specifically, for game $\mathcal{G}$ the independent actions of the UEs to set their power values should not only provide satisfactory individual solutions but should also mitigate potential interference levels in the network. Thereby, we design a payoff function that enables UEs to have lower data power levels while causing less interference in other UEs, given by:

$$\mu_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}) = \gamma_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}) + \lambda_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}), \quad (7)$$

where

$$\gamma_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}) = \frac{\sum_{\forall i \neq k} \rho_i \Big(\sum_{l \in \mathcal{M}_i} \beta_{i,l} \Big)^{\alpha}}{\rho_k \Big(\sum_{l \in \mathcal{M}_k} \beta_{k,l} \Big)^{\alpha}}, \quad (8a)$$

$$\lambda_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}) = \rho_k \Big(\sum_{l \in \mathcal{M}_k} \beta_{k,l} \Big)^{\alpha} \sum_{\forall i \neq k} \frac{1}{\rho_i \Big(\sum_{l \in \mathcal{M}_i} \beta_{i,l} \Big)^{\alpha}}, \quad (8b)$$

and α is an input parameter of game $\mathcal{G}$.

Particularly, the term $\gamma_k(\alpha, \boldsymbol{\rho})$ is based on the reciprocal of the SIR expression shown in [14, Section V, Equation (51)]. Initially, assuming the particular case where $\{\lambda_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)})\}_{\forall k} = 0$ and given α and $\boldsymbol{\rho}_{(-k)}$ fixed, it is easy to see that the best strategy or action for each UE exists and it would be to minimize the payoff function in (7) by choosing the highest possible value for the data power. However, we add the term $\lambda_k(\alpha, \boldsymbol{\rho})$ to the payoff function $\mu_k(\alpha, \boldsymbol{\rho})$ in order to make the decision of the UEs non-trivial and especially less selfish. In a general case, i.e., even if $\{\lambda_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)})\}_{\forall k} \neq 0$, it is possible to obtain a single value that minimizes (7) as it is a convex function in $\mathcal{P}_k, \forall k \in \mathcal{K}$. By solving $\partial \mu_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}) / \partial \rho_k = 0$, we can find the unique minimizer, ρ_k^*, of (7), as follows:

$$\rho_k^* = \sqrt{\left(\sum_{\forall i \neq k} \rho_i \Big(\sum_{l \in \mathcal{M}_i} \beta_{i,l} \Big)^{\alpha} \right) \left(\sum_{\forall i \neq k} \frac{\Big(\sum_{l \in \mathcal{M}_k} \beta_{k,l} \Big)^{2\alpha}}{\rho_i \Big(\sum_{l \in \mathcal{M}_i} \beta_{i,l} \Big)^{\alpha}} \right)^{-1}}. \quad (9)$$

From the point of view of the UEs, the term $\lambda_k(\alpha, \boldsymbol{\rho})$ in (7) represents a punishment for the UE who decides to excessively increase the value of the chosen data power. As a result, $\lambda_k(\alpha, \boldsymbol{\rho})$ can reduce interference levels in the system and avoid a greedy power allocation strategy, i.e., $\rho_k = P_{\max}$, $\forall k \in \mathcal{K}$.

In game theoretic approaches, we are interested in finding a Nash equilibrium as a solution. A Nash equilibrium is a strategy profile that satisfies the condition that no player can unilaterally improve its own payoff as shown in Definition 1:

Definition 1. *An ϵ-Nash equilibrium of parameterized game $\mathcal{G}(\alpha)$ is achieved when the payoff function $\mu_k(\alpha, \boldsymbol{\rho})$ is such that for all UE k:*

$$\mu_k(\alpha, \boldsymbol{\rho}) \leq \mu_k\big(\alpha, \rho_k, \boldsymbol{\rho}_{-(k)}\big) + \epsilon, \ \rho_k \in \mathcal{P}_k. \quad (10)$$

However, games may have a large number of Nash equilibrium points or may not have any. Generally, finding or even characterizing the set of these equilibrium points in terms of existence or uniqueness is a difficult task. Fortunately, there is a particular case of non-cooperative games called *potential games* for which the existence and uniqueness of a Nash equilibrium is ensured [15]. Basically, in a potential game the incentive of all players to change their actions can be expressed by a global payoff function called *potential function*. Mathematically, the proposed parameterized game $\mathcal{G}(\alpha)$ is a potential game if it complies with Definition 2 [5].

Definition 2. *If the proposed parameterized game $\mathcal{G}(\alpha)$ is a potential game, then there exists a function $u : \boldsymbol{\Upsilon} \to \mathbb{R}$ such that $\forall k \in \mathcal{K}$ and $\forall \rho_k, \rho'_k \in \mathcal{P}_k$:*

$$u(\alpha, \rho'_k, \boldsymbol{\rho}_{(-k)}) - u(\alpha, \rho_k, \boldsymbol{\rho}_{(-i)}) = \mu_k(\alpha, \rho'_k, \boldsymbol{\rho}_{(-i)}) - \mu_k(\alpha, \rho_k, \boldsymbol{\rho}_{(-k)}). \quad (11)$$

In this case, the function $u(\cdot)$ is called an exact potential function for the parameterized game $\mathcal{G}(\alpha)$ [5], [6].

Note that based on Definition 2, $\mathcal{G}(\alpha)$ is a potential game if it is possible to define a potential function, i.e., an UE-independent function that measures the same amount of change or marginal payoff for any unilaterally deviating UE. By exploiting the definition of payoff function in (7), we can prove that $\mathcal{G}(\alpha)$ is a potential game by showing that it has an exact potential function as explained in the following result:

Corollary 1. *There is an exact potential function for the parameterized game $\mathcal{G}(\alpha)$ and it is given by:*

$$u(\alpha, \boldsymbol{\rho}) = \frac{1}{2} \sum_{k \in \mathcal{K}} \Delta\mu_k(\alpha, \boldsymbol{\rho}). \quad (12)$$

Proof. This can be demonstrated with a relatively simple sequence of steps. Let $\rho_{\tilde{k}}, \rho'_{\tilde{k}} \in \mathcal{P}_{\tilde{k}}$ be two different and arbitrary data power values for a generic UE $\tilde{k}$. Suppose that UE $\tilde{k}$ changes its data power from $\rho_{\tilde{k}}$ to $\rho'_{\tilde{k}}$, then the change of its payoff function is: $\Delta\mu_{\tilde{k}} = \mu_{\tilde{k}}(\alpha, \rho'_{\tilde{k}}, \boldsymbol{\rho}_{(-\tilde{k})}) - \mu_{\tilde{k}}(\alpha, \rho_{\tilde{k}}, \boldsymbol{\rho}_{(-\tilde{k})})$. Moreover, the change of $u(\alpha, \boldsymbol{\rho})$ is: $\Delta u = u(\alpha, \rho'_{\tilde{k}}, \boldsymbol{\rho}_{(-\tilde{k})}) - u(\alpha, \rho_{\tilde{k}}, \boldsymbol{\rho}_{(-\tilde{k})})$. By developing the expressions $\Delta\mu_{\tilde{k}}$ and Δu, it is possible to show that: $\Delta\mu_{\tilde{k}} = \Delta u$, and thus $u(\alpha, \boldsymbol{\rho})$ is an exact potential function for the parameterized game $\mathcal{G}(\alpha)$. □

B. Proposed Iterative Algorithm

In order to develop a potential game-based approach to address the problem of power control for the uplink of cell-free massive MIMO systems, we propose a procedure where the power allocation is updated every iteration for each UE until reaching convergence. However, before specifically discussing this procedure, in order to achieve a practical implementation we define the following vector $\boldsymbol{\xi} \in \mathbb{R}^K$: $\boldsymbol{\xi} = [\xi_1, \cdots, \xi_k, \cdots, \xi_K]^{\mathrm{T}}$, where $\xi_k = \rho_k \Big(\sum_{l \in \mathcal{M}_k} \beta_{k,l} \Big)^{\alpha}$, $k \in \mathcal{K}$. Note that using the definition of $\boldsymbol{\xi}$, we can express the payoff function for each UE in terms of $\boldsymbol{\xi}$ only, as follows:

$$\mu_k(\boldsymbol{\xi}) = \frac{\sum_{\forall i \neq k} \xi_i}{\xi_k} + \xi_k \sum_{\forall i \neq k} \frac{1}{\xi_i}. \quad (13)$$

The complete procedure for the proposed power allocation strategy (PAS) based on game theory is described in Algorithm 1. First, in line 1, we can define $\boldsymbol{\rho}^{(0)}$ using any naive power allocation strategy, e.g., $\rho_k^{(0)} = P_{\max}/n$, $k \in \mathcal{K}$, where $n \in \mathbb{R}_+^*$. Moreover, the input parameter α must also be initialized, which is an important variable for the game as will be discussed in Session IV.

Algorithm 1 Game-based power allocation strategy (Game-PAS)

1: **Input:** Initialize $\boldsymbol{\rho}^{(0)}$, $\alpha \in \mathbb{R}$, $l \leftarrow 0$, and $\epsilon > 0$;
2: **Output:** Data power vector $\boldsymbol{\rho}$;
3: **loop**
4: UEs report the current data power to Master AP;
5: Master AP measures and broadcasts $\boldsymbol{\xi}^{(l)}$;
6: Find the payoff function using $\boldsymbol{\xi}^{(l)}$ and (13);
7: Find $\rho_k^\star$ making $\rho_k^\star \leftarrow \min\{\rho_k^*, P_{\max}\}$, where ρ_k^* is defined in (9);
8: **for** $k \in \mathcal{K}$ **do**
9: **if** $\mu_k(\rho_k^{(l)}, \boldsymbol{\rho}_{-(k)}^{(l)}) - \mu_k(\rho_k^\star, \boldsymbol{\rho}_{-(k)}^{(l)}) > \epsilon$ **then**
10: $\rho_k^{(l+1)} \leftarrow \rho_k^\star$;
11: **else**
12: $\rho_k^{(l+1)} \leftarrow \rho_k^{(l)}$;
13: **end if**
14: **end for** $l \leftarrow l + 1$;
15: **end loop when** $\rho_k^{(l)} = \rho_k^{(l-1)}$, $k \in \mathcal{K}$;

In the *outer loop* (lines 3-15) the following procedure is repeated: after information exchange between the UEs and the Master AP, each UE will independently choose a *best response* to the actions of the other UEs according to line 7. Then, in case of an improvement of the payoff function, each UE updates its data power, otherwise the current data power value is kept. Analogously to the best response approach in line 7, the power update also occurs individually, i.e., in the *inner loop* (lines 8-14) the power update for each UE is performed in a distributed way and thus it does not depend on K. Finally, Algorithm 1 ends when no UE can improve its payoff by unilateral deviation (cf. Definition 1).

C. Signaling and Convergence Analysis

During the execution of the proposed algorithm, an over-the-air signaling scheme must be considered for the two-way communication between the UEs and the Master AP, as performed in the lines 4 and 5 of Algorithm 1. Initially, each UE reports to the Master AP its current data power value. Next, the Master AP measures and then broadcasts $\boldsymbol{\xi}$. This is how each user receives the power allocation from other users.

Regarding the convergence of Algorithm 1, all finite potential games have the finite improvement path property [5]. Consequently, if every improvement path is finite and the best response approach provides an improvement of at least ϵ, then it must necessarily converge [5, See Chapter 5, Theorem 19].

IV. Simulation Results and Discussion

In this section, we conduct simulations relying on the setup introduced in [13]. In summary, the APs and UEs are independently and uniformly distributed in a 2 km × 2 km square. We apply the wrap-around technique to approximate an infinitely large network. Moreover, we assume that $\tau_{\mathrm{c}} = 200$, $\tau_{\mathrm{p}} = 10$, $\tau_{\mathrm{u}} = 190$, $P_{\max} = 100$ mW and a 20 MHz bandwidth.

A. Convergence Behavior

Fig. 1 shows the evolution of total power consumption of the UEs for different values of α versus the iterations of the game. For this result, we assume that initially all UEs transmit at their maximum power, i.e., $\rho_k^{(0)} = P_{\max}$, $k \in \mathcal{K}$. Thus, the total power usage at the beginning of the game is $K \cdot P_{\max} =$ 1000 mW. In particular, when $\alpha = 0$ note that no UE has an incentive to change the initial power allocation strategy and in that case the convergence of Algorithm 1 is immediate.

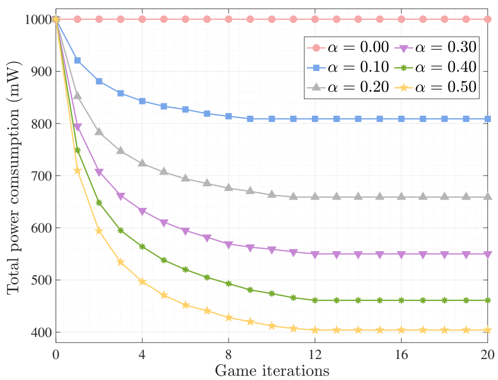

Fig. 1: Impact of α on the total power consumption versus iterations of the game assuming $L = 100$, $N = 4$ and $K = 10$.

On the other hand, in a more general case when $\alpha \neq 0$, the convergence behavior is different. As α increases, the strategy followed by the UEs converges to situations where the power expenditure is decreasing, i.e., the UEs are turned to a low power mode. As a result, this allows to improve the use of the energy stored in the battery and it shows that, interestingly, α has a direct impact on energy-saving. Obviously, as the values of α vary, we also obtain different data rates for the UEs and, therefore, non-trivial solutions especially in terms of energy efficiency (EE) defined as bit/J can be obtained.

B. Performance Comparison

In this section, the performance evaluation of the proposed power control is evaluated based on two aspects. First, we show the performance of Algorithm 1 in three different scenarios, namely, cell-free (discussed in Section II), small cell and massive MIMO systems. This is interesting as it shows the good adaptability of game theory-based approaches to various frameworks. In each scenario, we also consider a baseline scheme, in which each UE transmits at full power, i.e., we use the greedy power allocation strategy (Greedy-PAS) as benchmarking. It has been shown in the literature that this power allocation strategy can provide good SE and fairness [13]. Furthermore, we consider three different metrics: the total system SE, the minimum SE, and the total system EE, which is the sum of the EEs of each UE, defined as the ratio between the SE and the corresponding consumed power. Finally, our power control (Game-PAS) is performed for different α within the range $[0, 2]$ and the best performance for each metric is depicted.

Fig. 2 plots the total spectral efficiency versus the number of users. Specifically in this metric, the performance obtained by

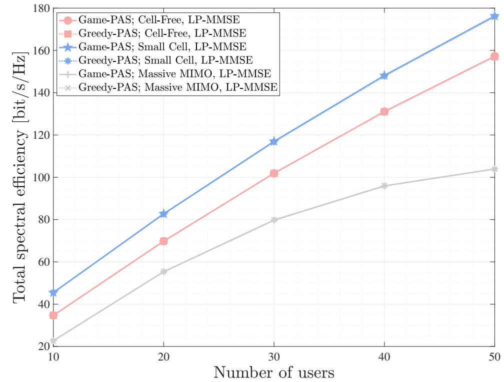

Fig. 2: Total spectral efficiency versus number of users assuming $L = 100$, $N = 4$ for the cell-free/small cell setups and $L = 4$, $N = 100$ for the massive MIMO case.

the proposed and baseline solutions are the same in all simulated setups. Basically, it means that from the point of view of total SE, and due to its simpler implementation, the Greedy-PAS solution has a better trade-off between performance and computational cost and is, therefore, the best option.

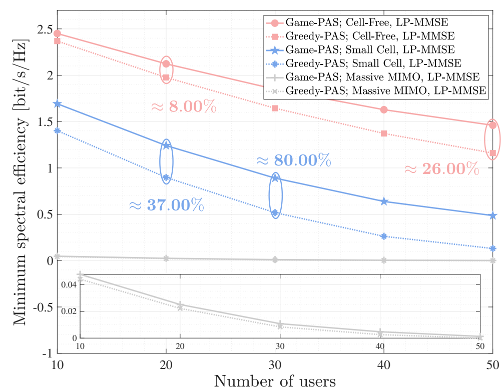

Fig. 3: Minimum spectral efficiency versus number of users assuming $L = 100$, $N = 4$ for the cell-free/small cell setups and $L = 4$, $N = 100$ for the massive MIMO case.

On the other hand, for the cell-free and small cell setups, significant performance gains in terms of minimum SE can be achieved using the proposed power control, as shown in Fig. 3. Moreover, we highlight that the gains tend to increase as the number of UEs increases. For the cell-free case, for example, we have average percentage gains around 8% and 26% when $K = 20$ and $K = 50$, respectively. Note that even more expressive gains of the proposed solution are obtained for the small cell case. In general, under interference-limited environments, as the number of UEs in the system increases, the power control problem becomes more relevant. However, trivial power allocation strategies usually neglect the impact of increasing UEs and, consequently, are ineffective in mitigating network interference by means of power control.

Finally, we plot the EE in Fig. 4. First, we highlight that the impact of α on the power usage shown in Fig. 1 has a direct effect in achieving enhanced EEs. Further, note that similarly to the minimum SE metric, the total EE performance gains also increase as K increases. This is particularly interesting as increasing the number of UEs in the network can rapidly

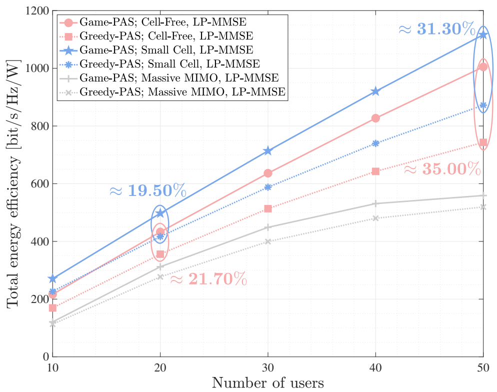

Fig. 4: Total energy efficiency versus number of users assuming $L = 100$, $N = 4$ for the cell-free/small cell setups and $L = 4$, $N = 100$ for the massive MIMO case.

lead to a growing concern with excessive energy demand, especially for the Greedy-PAS solution. At this point, energy efficient solutions are important and a more robust power allocation strategy such as the Game-PAS is critical to reduce the energy cost per transmitted bit and to improve the greenness of wireless systems.

C. Trade-off between EE and SE

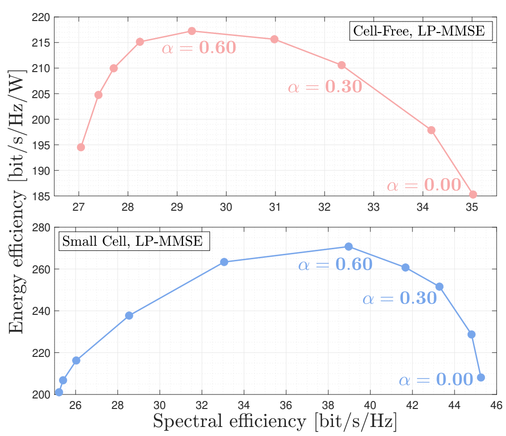

Fig. 5: Trade-off curve between EE and SE for the cell-free and small cell setups assuming $L = 100$, $N = 4$ and $K = 10$.

It is well-known that EE and SE are conflicting objectives and there exists an inherent trade-off between them. Thus, in the context of the proposed solution, it is interesting to show the impact of parameter α on the EE-SE trade-off. Fig. 5 presents the achieved SE and EE with different values of α for the cell-free and small cell setups. From the point of view of maximizing the SE, when $\alpha = 0.00$, as discussed in Fig. 1, the Game-PAS solution is equivalent to the Greedy-PAS solution and, in this case, the systems achieve high SE. However, as α increases, the EE is gradually improved until reaching a maximum value when $\alpha = 0.60$. Also, for other values of α, different solutions for EE and SE can be obtained. Therefore, Fig. 5 demonstrates that the proposed solution is efficient in achieving a flexible trade-off between EE and SE. For example, for small cells, when $\alpha = 0.30$, the EE metric has a gain around 20% with a small cost in terms of SE.

V. Final Comments

In this paper, we proposed a distributed game-theoretic method for power control in the uplink of cell-free systems. Simulations indicate that the proposed solution achieves significant performance gains in terms of minimum SE floor and power consumption with an improved EE. Moreover, by varying the α, we showed that it is possible to achieve different solutions for EE and SE. Hence, the proposed solution simplifies the process of joint optimization of these metrics and allows to obtain useful trade-offs between EE and SE.

VI. Acknowledgment

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